**Application of integration**

**Length of curve y=f(x)**

**Area under a curve y=f(x)**

**Surface and Volume formed by revolution of the curve y=f(x)**

**Length of curves y=f(x)**

1.

y=f(x)

B

y

Q(x+,y+)

P(x,y)

AB=s

PQ=

A

x=b

x=a

o

x

**Fig 7.**

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,

,

,

,

………….

………….

2. , parametric equations

,

,

,

,

,

3. polar equations

AB=s

PQ=

B

r=f()

y

Q(r+,+)

r+

P(r, )

r

A

+

x

o

**Fig 8**.

,

,

,

,

,

**Example 3.1:** Find the perimeter of the circle

y

(x,y)

O

x

x=0

x=a

**Fig 9**

= =

**Example 3.2:** Find the perimeter of the circle x=acos,y=asin

y

x

o

**Fig10**

=2

**Example 3.3:** Find the perimeter of the circle r=acos

r=acos

r=acos, r2=arcos, , , circle

r=acos

y

x

(a/2, 0)

O

**Fig11**

=

**Example 3.4:** Find the perimeter of the asteroid

Discussion x=acos,y=asin Parametric equation of circle

Cartesian equation of circle

x=acos,y=asin

y

x

O

**Fig.12**

x=acos, y=bsin Parametric equation of ellipse

Cartesian equation of ellipse

y

x

O

**Fig.13**

x=acos3,y=asin3 Parametric equation of asteroid

[ ]

Cartesian equation of asteroid

y

a

x

a

o

**Fig14**

x=acos3,y=bsin3 Parametric equation of hypocycloid

[ ]

Cartesian equation of hypocycloid

y

b

x

o

a

**Fig.15**

Discussion complete

Solution

y

a

a

x

o

**Fig 16**

=

**Example 3.5:** If s is the length of the arc of measured from the origin to the point (x,y) show that

Discussion about the curve: ,

If y=0 then x=0 and x=a that means the curve passes through the points (0,0) and (a,0)

Suppose if x=a/2 then

When x=2a then

y

(a/2,0.204a)

(a,0)

(0,0)

o

x

a

(a/2,-0.204a)

**Fig.17**

Solution

y

(x,y)

s

o

x

a

**Fig18**

,

,

===

==

=

**Intrinsic equation to a curve**

Def. If s denotes the length of the arc of curve measured from some fixed point A to a variable point p on the curve and if denotes the angle between the tangent at p and some reference line then a relation between s and is called an intrinsic equation to the curve. s and are called intrinsic co-ordinates.

**Intrinsic equation derived from cartesian equation**

y

s

p

A=

x

T

o

Let represent the equation of a curve. Now the length of the curve from A corresponding to is

----(1) and ------(2) where is the slop of the tangent at p with the x-axis. Then intrinsic equation can be obtain eliminate x and y from (1) and (2)

**Intrinsic equation derived from polar equation**

p

s

A

o

x

T

Let represent the polar equation of a curve. Now the length of the curve from A to any point is

----(1)

Let be the angle between the tangent and radius vector at and let be the angle made by the tangent at p with initial line . Then

--------(3) . The eliminant of from (1) , (2) and (3) is the required intrinsic equation of the curve.

Example: Find the intrinsic equation of the asteroid taking as fixed point.

Solution: Here ------(1)

-------(2)

-----(3)

From (1) and (2) we get

put in (3)

Ans.

Example: Find the intrinsic equation of the curve

and .

Solution: Here ------(1)

---------(2)

But Ans.

Example: Find the intrinsic equation of the cardiode

taking as fixed point.

Solution: Here ------(1)

---------(2)

But Ans.

The pedal equation of a circle is indicated by the formula p = r cos θ, where r stands for the radius, p stands for the perpendicular distance from the pedal point, and θ stands for the angle.

**Intrinsic equation derived from pedal equation**

Let be a pedal equation to the curve

------(1)

Also from differential calculus denoting the radius of the curvature ---------(2)

Eliminating r from (1) and (2) , we get relation when the write side is integrated , we get required intrinsic equation.

Example: Find the intrinsic equation of the curve

Solution:

--------(1)

Again

by (1)

Ans

**27-09-21 C 27-09-21 A 28-09-21 B recorded**

**Area under a curve y=f(x)**

1.

BA

y

y=f(x)

y



(x+,y+)

(x,y)

A

x

x=b

x=a

C

DA

o

**Fig19**

==ABCD

=

y=f(x)

y

y=d

(x+,y+)

x

(x,y)

y=c

o

x

2. ,

3.

B

r=f()

y

Q(r+,+)

r+

P(r, )

dA

r

A

+

x

o

**Fig 20**

=area of OAB

**29-09-21 C**

**Example 3.6:** Find the area of the circle

y

ydxx=a

(x,y)

y

dx

O

x

x=0

x=a

**Fig 21**

x=asin dx=acosd

**Example 3.7**: Find the area of the circle

y

x

o

**Fig 22**

=

Find the area of the cardioid

**Example 3.8:** Find the area of the circle x=acos, y=asin

y

x

o

**Fig 23**

**Example 3.9:** Find the area of the hypocycloid

y

ydxa

b

x

o

a

**Fig 24**

=

**Example 3.10:** Find the area of the loop of the curve

Similar to the figure 17 of with a=1

y

(a/2,0.204a)

(a,0)

(0,0)

o

x

a

(a/2,-0.204a)

**Fig. 25**

Solution

y

ydx

o

x

1

**Fig 26**

=

Can we apply the formula in this problem? We can not

**28-09-21A 29-09-21 B recorded**

**Example 3.11:** Find the area included between the curve and its asymptote. (Cissoid of Diocles)

Discussion

When x=0 y=0, the curve passes through (0,0)

When x=a

When x=a/2 , the curve passes through

y

o

x

a

**Fig. 27**

Solution

y

ydx

o

x

a

**Fig 28**

=

=

=

**04-10-21 C**

**Example 3.12:** Problem: Find the area of the following curve (Lemniscate of Bernoulli)

Discussion

When y=0, x=0, the curve passes through (0,0), (a,0), (-a,0)

When x=a/2 , the curve passes through

When x=-a/2 , the curve passes through

Solution

y

ydx

o

x

a

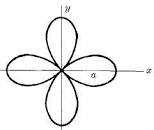
a

**Fig 29**



**=**

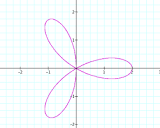
**Example 3.13:** Find the area of the loops of the curve



**Fig 30**

==

**Example 3.14:** Find the area of the loops of the curve



**Fig 31**

==

For Rose petals

r=acosn and r=asinn

if n is odd no. of loops is n

if n is even no. of loops is 2n.

**Example 3.13:** Find the area of the loops of the curve

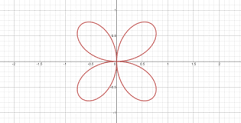
==

r=asin2

y

-

+



x

+

-

-

+

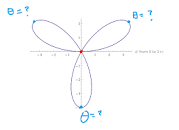
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-

**Example 3.14:** Find the area of the loops of the curve

==

r=asin3



**Example 3.15:** Find the area common to the circles  and

**04-10-21 A 05-10-21 B recorded**

y

**Area between two curves**

y1=f1(x)

y2=f2(x)

A

y1

y2

x=b

x=a

x

dx

o

**Fig. 32**

**Example 3.15:** Find the area bounded by the curve and the line

y

(4,8)

(0,0)

x

o

4

**Fig 33**

==

**06-10-21 C**

Find the area between the curve and the line

**Surfaces and Volumes of solids formed by the revolution of the curves y = f(x) about some axis**

y

x

o

y

S

V

Cone

x

o

y

x

o

y

V

S

x

o

y

x

o

y

S

Cylinderx

V

x

o

y

x

o

S

y

V

x

o

y

o

x

y

S

Sphere

V

o

x

y

x

O

y

S

V

x

O

y

S

V

x

O

y

V

S

x

O

**Formula for Volume formed by revolution of the curve about x-axis**

y

y=f(x)

(x+,y+)

(x,y)

y

x=a

o

x

x=b

**Fig.34**

y

One taka coin

Base area of the coin is 

height of the coin is

Total volume formed by the revolution of the curve about x-axis

**Formula for Volume formed by revolution of the curve about y-axis**

y=f(x)

y

y=d

(x+,y+)

x

(x,y)

y=c

o

x

**Fig 35**

Total volume formed by the revolution of the curve about y-axis

**Formula for the surface formed by revolution of the curve y=f(x) about x-axis**

y=f(x)

y

(x,y)

y

x

o

**Fig 36**

y

It will form a circular ring

Circumference of the circle is 

Breadth of the ring is





Total surface described by the curve

about x-axis

Similarly total surface described by the curve

about y-axis

**Example 3.16:** Find the volume and surface area of the solid formed by revolution of the circle about x-axis

=

y

o

x

**Fig 37**

, ,

**Example 3.17:** Find the volume of the solid produced by the revolution of the loop of the curve about the x-axis.

y

O

x=-a

x

a

**Fig 38**

**Discussion:**

When y=0, x=0, a therefore the curve passes through (0, 0) and

(a, 0)

When x=a/2

When x=-a

When x=-a/2

Solution



**05-10-21 A recorded 06- 10-21 B 11 10-21 C**

**Example 3.18:** The smaller of the two arcs into which the parabola divides the circle is rotated about the x-axis. Find the volume of the solid generated.

y

a, 2.83a)

x

a

(3a,0)

O

**Fig 39**

(x-a)(x+9a)=0 x=a, -9a accepted value is a

**Example 3.19:** The curve revolves round its asymptotes, Find its volume.

y

(x,y)

a-x

x



x

x=a

O

**Fig 40**

When x=0,y=0

When x=a

When x=a/2



=

=

=

= let

=

=

=

=

=

**Example 3.20:** Find the volume of the solid by revolution of the area bounded by and about the x-axis

y

(1, 3)

(0,0)

1

O

x

**Fig 41**

,

x=0, 1

y=0, 3

**Example 3.21:** The arc of the asteroid from to revolves about the x-axis. Show that the volume and surface area of the solid generated are respectively and

y

a

x

O

a

y

a

x

O

a

**Fig 42**

=



**11 10-21 A 12 10-21 B**

**End**

**Example 3.17:** Find the volume of the solid produced by the revolution of the loop of the curve about the x-axis.

y

O

x=-a

x

a

**Fig 38**

**Discussion:**

When y=0, x=0, a therefore the curve passes through (0, 0) and

(a, 0)

When x=a/2

When x=-a

When x=-a/2

**Solution**





**Example 3.17:** Find the volume of the solid produced by the revolution of the loop of the curve about the x-axis.



**Example 3.17:** Find the volume of the solid produced by the revolution of the loop of the curve about the y-axis.

**Example 3.17:** Find the volume of the solid produced by the revolution of the loop of the curve about the y-axis.

**Example 3.12:** Problem: Find the area of the following curve (Lemniscate of Bernoulli)

Discussion

When y=0, x=0, the curve passes through (0,0), (a,0), (-a,0)

When x=a/2 , the curve passes through

When x=-a/2 , the curve passes through

Solution

y

ydx

o

x

a

a

**Fig 29**



**=**

**Example 3.12:** Find the area of the following curve (Lemniscate of Bernoulli)

**Example 3.11:** Find the area included between the curve and its asymptote. (Cissoid of Diocles)

Discussion

When x=0 y=0, the curve passes through (0,0)

When x=a

When x=a/2 , the curve passes through

y

o

x

a

**Fig. 27**

Solution

y

ydx

o

x

a

**Fig 28**

=

=

=

**Example 3.11:** Find the area included between the curve and its asymptote.